LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2019**

PST 1503 – STATISTICAL MATHEMATICS

Date: 05-11-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

SECTION – A

Answer ALL questions. Each carries TWO marks. (10 x 2 = 20 marks)

1. Give the definition of sequence of real numbers with an example.
2. Show that the sequence (n) where n $\in$ N does not have a limit.
3. Let L be the limit of a sequence (s_n). Then prove that every open interval containing L contains all but a finite number of terms of the sequence.
4. Show that the series 1 + 2 + 3 + ... + n + ... is divergent.
5. State necessary and sufficient condition for the convergence of a telescoping series.
6. Obtain sup f(x) and inf f(x) for the function $f(x) = e^{-|x|}$ on $(-\infty, \infty)$.
7. Show that the function $f(x) = x^2$ is bounded on each bounded interval of R, but is unbounded on R.
8. If $f(x) = |x|$ for $x \in (-\infty, \infty)$, then prove that ‘f’ is continuous at ‘0’, but not differentiable at ‘0’.
9. Give the Definition of an upper and lower integral of a bounded function ‘f’ over [a, b].
10. Explain linear independence of ‘k’ vectors with an illustration.

SECTION – B

Answer any FIVE questions. Each carries EIGHT marks. (5 x 8 = 40 marks)

11. Give the formula for nth term of the sequence (s_n) = (1, -4, 7, -10, 13, ...) . Check if each of the following sequence is a subsequence of (s_n):
    (i) (1, 7, 13, ...)
    (ii) (-4, -10, -16, ...)
    (iii) (7, -4, 13, -10, ...)
    (iv) (4, 10, 16, ...).
12. If $s_n = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{n+n}$, then verify if (s_n) converges.
13. Check if the limit of the sum of two convergent sequences is the sum of their limits.
14. Verify the convergence of the series $\sum_{n=0}^{\infty} x^n$ when (i) 0 < x < 1, and (ii) x ≥ 1.
15. Prove that absolute convergence of $\sum a_n$ implies convergence of $\sum a_n$. Verify if the converse is true.
16. Verify the convergence of a series which is dominated by an absolutely convergent series.
17. Let σ be the partition $\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$ of [0, 1]. Let f(x) = x defined on [0, 1]. Obtain U [f; σ] and L[f; σ].
18. List any four properties of the Riemann integral.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. (2 x 20 = 40 marks)

19(a). Examine the convergence of the sequence $((1 + \frac{1}{n})^n)$. (10)

19(b). Show that a monotonic series converges if and only if it is bounded. (10)

20. Let $f(x) = \frac{1}{x}$. Then prove the following:

(i) f is unbounded for $0 < x < \infty$

(ii) $\inf_{x > 0} f(x) = 0$

(iii) f is bounded on (a, ∞) for any $a > 0$. (20)

21(a). Write the definition of improper integral of the first, second and third kind with an example for each kind. (8)

21(b). Check the convergence of the following integrals:

(i) $\int_1^\infty \frac{1}{x^2}\, dx$ (ii) $\int_0^\infty e^{-x} dx$ (iii) $\int_1^\infty \frac{1}{\sqrt{x}} dx$. (12)

22(a). State Taylor's formula and Maclaurin's theorem with Lagranges form of remainder. Hence find Taylor's formula for f(x) = log(1 + x) about a = 2 and n = 4. (10)

22(b). Discuss the characteristic value problem and define the characteristic roots and vectors. Hence obtain the characteristic roots and vectors for the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. (10)

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