LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **NOVEMBER 2019**

ST 3506 – MATRIX AND LINEAR ALGEBRA

Date: 31-10-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

SECTION – A : Answer ALL the questions **(10X 2 = 20)**

1 Define symmetric matrix with an example.

2 When do you say that the matrix is singular and non-singular?

3 Compute the rank of the matrix A = $\begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix}$.

4 Mention any two properties of determinant.

5 When do we say that the vectors $X_1, X_2 \ldots, X_r$ are linearly dependent?

6 Define linear transformation.

7 How do you define vector space?

8 Explain linear homogeneous equations.

9 Find the characteristic root of the matrix $\begin{bmatrix} 4 & 3 \\ 6 & 5 \end{bmatrix}$

10 Show that if λ is a characteristic root of a matrix A, then prove that λ^k is the characteristic root of A^k.

SECTION – B: Answer ANY FIVE questions **(5X 8 = 40)**

11 Prove that if A and B are symmetric matrices, then AB is symmetric if and only if AB = BA.

12 Find the rank of the matrix A.

$$\begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 3 & 1 & 1 & 3 \end{bmatrix}$$

13 Find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 6 & 7 \end{bmatrix}$$

14 Under what condition the rank of the following matrix A is 3? Is it possible for the rank to be 1? Why?

$$\begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & x \\ 1 & 0 & 0 \end{bmatrix}$$

15 Show how the product of two matrices is related to the composition of Linear Transformations

16 Explain Cramer’s rule with an example.

17 Show that the set of 3 vectors X1 = (1 0 0), X2 = (0 1 0) and X3 = (0 0 1) are linearly independent.

18 Find the Eigen values and the corresponding Eigen vector of the following matrix

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

SECTION – C: answer ANY TWO questions **(2X 20= 40)**

19 a) State and prove Cayley-Hamilton theorem. (10)

b) using Cayley-Hamilton theorem, find the inverse of $\begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & 0 \\ 3 & -1 & 0 \end{bmatrix}$ (10)

20 a) Using Cramer's rule find the solution of (12)

$2x - y + 3z = 9$

$x + y + z = 6$

$x - y + z = 2$

b) Write any four properties of Eigen values and Eigen vectors (8)

21 a) Show that $\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} = (a-b)(b-c)(c-a)$. (10)

b) Find the rank of the matrix $A = \begin{bmatrix} 5 & 8 & 7 & 5 \\ 3 & 2 & 1 & 3 \\ 2 & 4 & 3 & 2 \\ 1 & 2 & 3 & 0 \end{bmatrix}$ (10)

22 a) Solve for x $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$ (10)

b) Show that every square matrix with complex elements can be expressed uniquely as the sum of a Hermitian and a Skew- Hermitian Matrix. (10)

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